Code No. : 11011M SP

VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD
B.E. I Year I-Semester Special Examination, September-2017
Mathematics - I

Time: **3** hours

Max. Marks: **50**

*Note: Answer **ALL** questions in **Part-A** and any **FIVE** from **Part-B***

Part-A (15 Marks)

1. Define Rank of a Matrix. [1]

2. If a series $\sum u_n$ is divergent, then find $\lim_{n \to \infty}(u_n - u_{n-1})$. [1]

3. Find the Taylor's series expansion of e^x about x = 1. [1]

4. Find Jacobian of $x = r\cos\theta$, $y = r\sin\theta$. [1]

5. Represent the area of a rectangle formed by coordinate axes and the lines x = 5 and y = -4 as a double integral. [1]

6. If the characteristic equation of a matrix is $3\lambda^2 + 5\lambda - 8 = 0$ then find the Eigen values of that matrix. [2]

7. Give example for (i) Conditionally convergent series (ii) Absolutely convergent series. [2]

8. Find the circle of curvature for the curve $y = -x^2 + x + 1$ at (1, 1). [2]

9. If $z = u^2 + v^2$, $u = a\cos t, v = b\sin t$ find dz/dt. [2]

10. Evaluate $\int_0^2 \int_1^x \int_0^y z\,dz\,dy\,dx$. [2]

Part-B (5 × 7 = 35 Marks)

11. a) Find the Eigen values and Eigen Vectors of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ [3]

   b) Using Cayley-Hamilton Theorem and find A^{-1}, where $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$ [4]

12. a) Examine the convergence or divergence of $\dfrac{2 \cdot 1}{3^2 \cdot 4^2} + \dfrac{4 \cdot 3}{5^2 \cdot 6^2} + \dfrac{6 \cdot 5}{7^2 \cdot 8^2} + \cdots\cdots$ [3]

   b) Test whether the series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\sqrt{n+1} - \sqrt{n}\right)$ is Absolutely or Conditionally convergent. [4]

13. a) Find the Evolutes of the tractrix $x = a\left(\cos t + \log\tan\frac{t}{2}\right), y = a\sin t$. [4]

   b) Find the radius of curvature at the origin of the curve $y^2 = x^2 \dfrac{a+x}{a-x}$ [3]

14. a) Find the maximum /minimum values of $f = x^2y^2 - 5x^2 - 8xy - 5y^2$. [4]

   b) If $u(x, y) = x^2 Tan^{-1}\left(\dfrac{y}{x}\right) + y^2 Tan^{-1}\left(\dfrac{x}{y}\right), x > 0, y > 0$ then find $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y}$. [3]

15. a) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$. [4]

    b) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+y} e^{x+y+z} dxdydz$. [3]

16. a) Reduce the quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to canonical form by by orthogonal transformation. [4]

    b) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n.3^n}$ $(x>0)$. [3]

17. Answer any *two* of the following:

    a) Show that the Evolute of the ellipse $x = a\cos\theta; y = b\sin\theta$ is $(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}} = (a^2 - b^2)^{\frac{2}{3}}$. [3½]

    b) Find the point on the sphere $x^2 + y^2 + z^2 = 1$, nearest to the point (2, 1, 1) using method of Lagrange's multipliers. [3½]

    c) Evaluate $\iint_R (x+y) dydx$, R is the region bounded by $x = 0, x = 2, y = x, y = x+2$. [3½]

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